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The electric dipole moment of the nucleons in holographic QCD

Deog Ki Hong,[a] Hyun-Chul Kim[ab] and Sanjay Siwach[a]

[a] *Department of Physics, Pusan National University, Busan 609-735, Korea*
[b] *Nuclear Physics & Radiation Technology Institute (NuRI), Pusan National University, Busan 609-735, Korea*
E-mail: dkhong@pusan.ac.kr, hchkim@pusan.ac.kr, sksiwach@hotmail.com

Ho-Ung Yee

School of Physics, Korea Institute for Advanced Study, Seoul 130-012, Korea
E-mail: ho-ung.yee@kias.re.kr

Abstract: We introduce the strong CP-violation in the framework of AdS/QCD model and calculate the electric dipole moments of nucleons as well as the CP-violating pion-nucleon coupling. Our holographic estimate of the electric dipole moments gives for the neutron $d_n = 1.08 \times 10^{-16}\ \bar{\theta}\, e \cdot \mathrm{cm}$, which is comparable with previous estimates. We also predict that the electric dipole moment of the proton should be precisely the minus of the neutron electric dipole moment, thus leading to a new sum rule on the electric dipole moments of baryons.

Keywords: AdS-CFT Correspondence, Phenomenological Models, QCD.

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Contents

1. Introduction

The strong interactions of elementary particles are known to be highly symmetric. The most stringently tested global symmetry of strong interactions in the framework of relativistic quantum field theory is CP, charge conjugation (C) times parity (P), or the time reversal symmetry, T. The experimental upper bound on the CP violation comes from the absence of the electric dipole moment (EDM) of neutron,

$$|d_n| < 2.9 \times 10^{-26}\, e \cdot \mathrm{cm} \tag{1.1}$$

with 90 % confidence level [1]. More precise measurements of the neutron EDM are now under way to improve the current limit by a factor of 50 to 100 [2]. On the other hand, the standard theory of the strong interactions, QCD, allows a CP violating term, called the θ term,

$$\mathcal{L}_{\mathrm{QCD}} \ni \frac{\theta}{64\pi^2}\, \epsilon_{\mu\nu\rho\sigma} G^{a\mu\nu} G^{a\rho\sigma}\,, \tag{1.2}$$

where $G^a_{\mu\nu}$ is the field strength tensor of gluons.

The quark mass term in the QCD Lagrangian,

$$\mathcal{L}_m = -\bar{q}^i_L (M_q)_{ij} q^j_R + \mathrm{h.c.} \quad , \tag{1.3}$$

shifts θ by a chiral rotation, $\bar{\theta} = \theta + \mathrm{Arg\,Det} M_q$, which is the physical strong CP-violation angle. The $\bar{\theta}$ term contributes to the neutron electric dipole moment [3−9]

$$d_n = c \times 10^{-16}\, \bar{\theta}\, e \cdot \mathrm{cm}\,, \tag{1.4}$$

where c is a constant of order one, as can be seen from the naive dimensional analysis,

$$d_n \sim \left(\frac{1}{m_N}\right) \left(\frac{m_q\, \bar{\theta}}{m_N}\right) \tag{1.5}$$

with m_N and m_q being the masses of nucleons and quarks, respectively. The θ parameter therefore has to be extremely fine-tuned to be consistent with the experimental data,

$$\theta + \mathrm{Arg}\,\mathrm{det} M \lesssim 10^{-9}\,. \tag{1.6}$$

Such fine-tuning is known as the strong CP problem and several solutions are proposed to solve the strong CP problem [10].

Since not only QCD but the electroweak interactions [11] and also the physics beyond the standard model contribute to the neutron electric dipole moment, it is quite important to estimate the QCD contribution accurately. In this letter we estimate the electric dipole moment of nucleons as well as the CP-violating pion-nucleon coupling in holographic models of QCD, which have been quite successful in describing the properties of hadrons. We find that our holographic estimate of nucleon EDM is comparable with previous results, based on lattice calculations [7], current algebra [3 − 5], chiral perturbation theory [8] or QCD sum rule [9]. We also get an interesting sum rule for the nucleon EDM's; the EDM of neutron is opposite to that of proton, $d_n + d_p = 0$, which is consistent with the recent lattice result [7] and the estimate from the Light-Front formalism [12].

The new sum rule on EDM is insensitive to any higher order corrections in $1/N_c$ and is also a model-independent prediction of holographic QCD, where baryons are realized as instanton solitons. As was shown in [13], the Pauli term in the 5D action should not have any U(1) coupling, since the instanton has only SU(2) nonabelian tails, and thus the anomalous magnetic moments of baryons should add up to zero for each flavor multiplets. The same should hold for the electric dipole moments, because they are related to the anomalous magnetic moments by a U(1) axial rotation.

2. The model with baryons

A holographic model for spin $\frac{1}{2}$ baryons is constructed for two flavors in [14].[1] In this section, we briefly summarize the model [14, 16], since we will be studying the nucleon EDM in the framework of this model, closely following the notations. For the meson sector, we take the simplest hard-wall AdS/QCD model as in [17, 18], with the metric

$$ds^2 = \frac{1}{z^2}\left(-dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu\right) \quad , \tag{2.1}$$

where $0 \leq z \leq z_m$ and $\eta_{\mu\nu} = \mathrm{diag}(+1, -1, -1, -1)$. This model captures the important aspects of low energy chiral dynamics of light mesons, especially that of pions and vector mesons. One should think of this type of holographic models as alternative effective theories of strongly coupled field theory like low-energy QCD in the large N_c limit. The theory is expected to have a classical nature in the large N_c approximation. In conjunction with the renormalization group invariance, which is an essential element of quantum field theories, the resulting large N_c classical master field should develop a new, dynamically generated, space, which corresponds to the energy scale or the renormalization scale of the Wilsonian

[1] See ref. [15] for a model of higher spin Regge trajectory.

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type. The large N_c classical nature and the Wilsonian renormalization group fit together in the extra dimension. The low-lying spin $\frac{1}{2}$ baryons such as a proton-neutron isospin doublet and its excitations are shown to be naturally realized as 5D Dirac spinors in this picture [14].

Since we are interested in the spin $\frac{1}{2}$, isospin $\frac{1}{2}$ baryons, we introduce (Dirac) spinors in our 5 dimensional AdS slice as a holographic realization of spin $\frac{1}{2}$ baryons.[2] However, there are two caveats we have to be careful about. The first one is the representation of our 5D holographic baryon fields under the chiral symmetry $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ of QCD, which becomes a gauge symmetry in the dual 5D model. We know that the lowest-lying 4D excitations, the nucleons, form a doublet under the isospin $\mathrm{SU}(2)_I$, the diagonal part of $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$, after chiral symmetry breaking, but there is no unique way to assign the nucleon charges under the original $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ symmetry, since the Nambu-Goldstone fields can be always multiplied to the nucleon fields [19].

For $N_F = 2$ case however, there is an answer. To match the UV anomaly of $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_B$ from massless chiral quarks (u_L, d_L) and (u_R, d_R) with $N_c = 3$, there must exist massless chiral baryon doublets (p_L, n_L) and (p_R, n_R) with the representations $(\Box, 1)$ and $(1, \Box)$ under $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ respectively, when the theory is confining but in the chirally symmetric false vacuum [20]. Their baryon charge is $N_c = 3$ times the quark baryon charge.

Chiral symmetry breaking in the true vacuum introduces a mass coupling for nucleons,

$$\mathcal{L}_{\chi SB} \sim -m_N \begin{pmatrix} \bar{p}_L \\ \bar{n}_L \end{pmatrix} \Sigma \begin{pmatrix} p_R & n_R \end{pmatrix} + \mathrm{h.c.} \quad , \tag{2.2}$$

where $\Sigma = \exp(\frac{2i\pi}{f_\pi}) \in \mathrm{SU}(2)$ is the non-linear group field in the broken phase, which transforms non-linearly as $\Sigma \to U_L \, \Sigma \, U_R^\dagger$ under $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$. As $\langle \Sigma \rangle = \mathbf{1}$ in the true vacuum, the above is invariant under the isospin $\mathrm{SU}(2)_I$ for which nucleons form a doublet. In the symmetric (false) vacuum, Σ and the isospin singlet the sigma meson (σ) will be completed to a linear field X which is bi-fundamental $(\Box, \bar{\Box})$ under $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$, and we should have the following term in the theory

$$\mathcal{L}_m = -g \begin{pmatrix} \bar{p}_L \\ \bar{n}_L \end{pmatrix} X \begin{pmatrix} p_R & n_R \end{pmatrix} + \mathrm{h.c.} \quad , \tag{2.3}$$

to have the coupling (2.2) in the broken phase $\langle X \rangle \sim \Lambda_{\mathrm{QCD}} \mathbf{1}$.

Based on the above consideration, we find the simplest choice is to introduce two 5D spinors N_1 and N_2, of representation $(\Box, 1)$ and $(1, \Box)$ respectively under $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ 5D gauge symmetry [14]. Upon the Kaluza-Klein (KK) reduction to 4D, the modes from N_1 and N_2 must include the above-mentioned massless chiral baryon excitations (p_L, n_L) and (p_R, n_R) respectively, in the unbroken chiral-symmetric limit. This requirement uniquely fixes the IR boundary conditions for N_1 and N_2 at $z = z_m$. Then, the natural holographic realization of the nucleon mass coupling (2.3) is to introduce a gauge invariant 5D

[2]Note that a 5D spinor has 4-components, like a Dirac spinor in 4D.

interaction

$$\mathcal{L}_{5\mathrm{D}} \;=\; -g\,\bar{N}_1 X N_2 \;+\; \mathrm{h.c.}\,, \tag{2.4}$$

where X is the bi-fundamental 5D scalar field of $(\square, \bar{\square})$, whose vacuum expectation value (VEV) breaks the chiral symmetry in the model. The above coupling will induce the mass coupling (2.2) between the would-be massless 4D chiral baryons from N_1 and N_2 in the chiral-symmetry broken vacuum $X \sim \frac{1}{2}\Sigma z^3$. The coupling strength g must then be fitted to reproduce the nucleon mass $m_N = 0.94\,\mathrm{GeV}$ as the lowest mass eigenvalue.

The other caveat in our holographic baryon model is the question of chirality in the 5 dimensional context. Though N_1 (N_2) is the holographic dual field to the 4D left(right)-handed nucleon operator,[3] there is no chirality in 5 dimensions. The 4D chirality is in fact encoded in the sign of 5D Dirac mass term [21]. For a positive 5D mass,[4] only the right-handed component of the 5D spinor survives near the boundary $z \to 0$, and this acts as a source for the left-handed chiral operator in 4 dimensions. The story is simply reversed for the opposite sign case. The magnitude of the 5D mass is given by the AdS/CFT relation

$$m_5^2 \;=\; (\Delta - 2)^2\,, \tag{2.5}$$

where we take $\Delta = \frac{9}{2}$ for a composite baryon operator of three quarks. Considering the anomalous dimension of the composite operator might lead to slightly different results.

To wrap up the above discussions, our 5 dimensional holographic model for spin $\frac{1}{2}$, isospin $\frac{1}{2}$ baryons in $N_F = 2$ sector is given by

$$\begin{aligned} S_{\mathrm{kin}} &= \int dz \int dx^4 \sqrt{G_5}\left[i\bar{N}_1\Gamma^M D_M N_1 + i\bar{N}_2\Gamma^M D_M N_2 - \frac{5}{2}\bar{N}_1 N_1 + \frac{5}{2}\bar{N}_2 N_2\right]\,, \\ S_m &= \int dz \int dx^4 \sqrt{G_5}\left[-g\bar{N}_1 X N_2 - g\bar{N}_2 X^\dagger N_1\right]\,, \end{aligned} \tag{2.6}$$

where D_M is the gauge and Lorentz covariant derivative, $\sqrt{G_5} = \frac{1}{z^5}$, and the gamma matrices in our AdS_5 are related to the 4D gamma matrices as $\Gamma^\mu = z\gamma^\mu$ for $\mu = 0, 1, 2, 3$ and $\Gamma^5 = -iz\gamma^5$. Upon KK reduction to 4D, it is easy to find the eigenmode equations for the mass spectrum of 4D spin $\frac{1}{2}$ baryons. Writing $N_1(x, z) = f_{1L}(z)B_L(x) + f_{1R}(z)B_R(x)$ and similarly for $N_2(x, z) = f_{2L}(z)B_L(x) + f_{2R}(z)B_R(x)$, where $B_{L,R}$ are the components of the 4D eigenmode spinor $B = (B_L, B_R)^T$ with mass m_N to be determined, we have

$$\begin{aligned} \begin{pmatrix} \partial_z - \frac{\Delta}{z} & -\frac{g\langle X\rangle}{z} \\ -\frac{g\langle X^\dagger\rangle}{z} & \partial_z - \frac{4-\Delta}{z} \end{pmatrix} \begin{pmatrix} f_{1L} \\ f_{2L} \end{pmatrix} &= -m_N \begin{pmatrix} f_{1R} \\ f_{2R} \end{pmatrix}\,, \\ \begin{pmatrix} \partial_z - \frac{4-\Delta}{z} & \frac{g\langle X\rangle}{z} \\ \frac{g\langle X^\dagger\rangle}{z} & \partial_z - \frac{\Delta}{z} \end{pmatrix} \begin{pmatrix} f_{1R} \\ f_{2R} \end{pmatrix} &= m_N \begin{pmatrix} f_{1L} \\ f_{2L} \end{pmatrix}\,, \end{aligned} \tag{2.7}$$

with Δ ($= \frac{9}{2}$ in our case) in general. As mentioned before, the existence of the chiral zero modes when $\langle X\rangle = 0$ requires us the IR boundary condition $f_{1R}(z_m) = f_{2L}(z_m) = 0$. In

[3] We are using the same notation for both the operator and the state of the 4D chiral baryons without confusion. Strictly speaking, N_1 is dual to the left-handed chiral operator made of three left-handed quarks, and vice versa for N_2.

[4] Our convention is $\mathcal{L}_m = -m_5\bar{N}N$ and $\Gamma^5 = (-i)z\gamma^5$ with $\gamma^5\psi_L = +\psi_L$.

the meson sector the best fit was found for $\langle X\rangle = \frac{1}{2}(m_q z + \sigma z^3)$ with $m_q = 2.34\,\mathrm{MeV}$, $\sigma = (311\,\mathrm{MeV})^3$, and the IR cut-off $z_m = (330\,\mathrm{MeV})^{-1}$ [17, 18]. Then, the only remaining parameter of the theory is the dimensionless coupling g, which was found to be $g = 9.18$ in [14] to reproduce $m_N = 0.94\,\mathrm{GeV}$ as a lowest mass eigenvalue.

In [13], which discussed nucleons in the top-down Sakai-Sugimoto model, an important new operator, responsible for anomalous magnetic dipole moments, among others, was identified, which will also be important for our analysis of CP-violating electric dipole moments. Experimentally, the proton magnetic moment is $\mu_p = 2.8\mu_N = \mu_N + 1.8\mu_N$ where the Nuclear Magneton $\mu_N = \frac{e}{2m_N}$ is from the minimal coupling of charge 1, while the neutron has $\mu_n = -1.8\mu_N$. It is clear that the anomalous piece has a structure of isospin doublet without $\mathrm{U}(1)_B$ charge, as shown in [13]. Since electromagnetic coupling is a sum of the isospin and $\mathrm{U}(1)_B$, $Q = \frac{1}{2}B + I_3$, the operator responsible for the anomalous magnetic moment does not include $\mathrm{U}(1)_B$.

It is easy to find the corresponding 5D operator which induces the desired anomalous magnetic moment upon 4D reduction,

$$\mathcal{L}_{\mathrm{dipole}} = iD\left[\bar{N}_1\Gamma^{MN}(F_L)_{MN}N_1 - \bar{N}_2\Gamma^{MN}(F_R)_{MN}N_2\right] \quad , \tag{2.8}$$

where $F_{L,R}$ are the field strengths of $A_{L,R}$ and D is a real parameter that must be fixed to reproduce $\mu_{\mathrm{anomalous}} = 1.8\mu_N$.[5] Note that $A_{L,R}$ in our model do not include $\mathrm{U}(1)_B$, as required. In fact, this absence of $\mathrm{U}(1)_B$ in the 5D effective operator (2.8) has its origin from the fact that baryons in AdS/QCD arise as instantonic solitons of small size in our (4+1)-dim gauge theory of $A_{L,R}$. As we treat them as point-like with the effective fields N_1 and N_2, we need to take into account their long-range instanton tail of $A_{L,R}$, and the operator (2.8) exactly sources those tail profiles. In a more complete description, the coefficient D would be determined by the stabilized size of small instanton-solitons, whereas here it is a fitted parameter against experiments. Since instanton profiles are purely non-abelian, the operator (2.8) should not include $\mathrm{U}(1)_B$, which leads to a model-independent sum rule for any quantities derived from the operator such as the anomalous magnetic moments.

3. Physics of vacuum alignment

The strong CP-violation in QCD can be introduced either in the vacuum θ-angle or as complex phases in the quark mass matrix M_q defined as

$$\mathcal{L}_m = -\bar{q}^i_L(M_q)_{ij}q^j_R + \mathrm{h.c.} \quad . \tag{3.1}$$

As is well-known, the anomalous axial $\mathrm{U}(1)_A$ rotation can shift the θ-angle to zero, making the strong CP-violation appear only in M_q or vice versa. The physical strong CP-violation angle is[6]

$$\bar{\theta} = \theta + \mathrm{Arg}\,\mathrm{Det}(M_q) \quad . \tag{3.2}$$

[5]In [13], D was reliably predicted from string theory, which explains $\mu_{\mathrm{anomalous}} = 1.8\mu_N$ quite well. But in our bottom-up model it is a fitting parameter, though the relative minus sign in (2.8) is dictated by 4D parity invariance, as easily seen from the fact that 5D Dirac mass term flips its sign under 4D parity.

[6]The θ-angle is normalized as $\mathcal{L}_\theta = \frac{\theta}{32\pi^2}\mathrm{Tr}(F\wedge F)$ with $\frac{1}{32\pi^2}\int \mathrm{Tr}(F\wedge F) = 1$ for a single QCD instanton.

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Presumably, the θ-angle can be described in the holographic model by a kind of axion [22], but for our purpose it is much more convenient to work in the frame where the strong CP-violation appears in M_q only. This is because in our AdS/QCD model, M_q is easily described by the non-normalizable mode of the scalar X,

$$\langle X(z)\rangle = \frac{1}{2}M_q z + \frac{1}{2}\Sigma z^3\,, \tag{3.3}$$

where Σ is the quark bi-linear condensate $(\Sigma)^{ij} = -\left\langle \bar{q}_R^j q_L^i \right\rangle$.

In the CP-symmetric case of $M_q = \mathrm{diag}(m_u, m_d)$ with small real masses $m_u, m_d \ll \Lambda_{\rm QCD}$, it looks natural to have the bi-quark condensate proportional to identity matrix[7]

$$\Sigma \sim \Lambda_{\rm QCD}^3 \mathbf{1}_2 \quad , \tag{3.4}$$

as was used before and will be shown rigorously in a moment. However, as we turn on small CP-violation as phases $\alpha_{1,2}$ of quark mass $M_q = \mathrm{diag}(e^{i\alpha_1}m_u, e^{i\alpha_2}m_d)$, it is not obvious which polarization Σ will take in the SU(2) space of its moduli, $\Sigma = \Lambda^3 U$ with $U \in \mathrm{SU}(2)$ and Λ is a scale proportional to $\Lambda_{\rm QCD}$, the intrinsic scale of QCD. This is the problem of vacuum alignment, first considered by Dashen [23].

The vacuum moduli of Σ form a coset space $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R / \mathrm{SU}(2)_I$. Since the quark mass M_q breaks the chiral symmetry, it lifts the moduli space by inducing a potential,

$$V_m = -\mathrm{Tr}(M_q \Sigma^\dagger) + \mathrm{h.c.} \quad , \tag{3.5}$$

which can be inferred from (3.1). With our diagonal mass matrix M_q, it is natural to make $\langle\Sigma\rangle$ align along its diagonal direction by a $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ chiral transformation, that is,

$$\langle\Sigma\rangle = \Lambda^3 \begin{pmatrix} e^{ix} & 0 \\ 0 & e^{-ix} \end{pmatrix} \quad , \tag{3.6}$$

to get the vacuum potential $V_m = -2m_u\,\Lambda^3\,\cos(x-\alpha_1) - 2m_d\,\Lambda^3\,\cos(x+\alpha_2)$. For small $\alpha_{1,2}$, related to $\bar\theta \sim 10^{-9}$, as will be seen in a moment, we expand the above cosines to get $V_m/\Lambda^3 \approx (m_u+m_d)x^2 - 2(m_u\alpha_1 - m_d\alpha_2)x + \mathrm{const.}$ The potential minimizes at

$$x = \frac{m_u\alpha_1 - m_d\alpha_2}{m_u + m_d} \quad , \tag{3.7}$$

as the aligned vacuum polarization in the presence of the strong CP-violation. It also confirms that identity matrix is the ground state for the CP-symmetric case, $\alpha_{1,2} = 0$.

For our analysis, it is more convenient to take a non-anomalous $\mathrm{SU}(2)_A$ axial rotation to remove the phase angle x in Σ.[8] This brings the quark mass angles to $\alpha_1 \to \alpha_1 -$

[7] Isospin violating effects from $m_u \neq m_d$ to the bi-quark condensate can be neglected to first order in m_q, as we are interested in the results that are first order in m_q and $\bar\theta$.

[8] Note that this global rotation also rotates M_q simultaneously. When we discussed the lifted moduli of SU(2) above, we fixed M_q. In other words, only the relative $\mathrm{SU}(2)_A$ angle between M_q and Σ is physical and we can always go to the frame where $\Sigma \sim \mathbf{1}$, as the coset space is homogeneous.

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$x = \frac{(\alpha_1+\alpha_2)m_d}{m_u+m_d}$ and $\alpha_2 \to \alpha_2 + x = \frac{(\alpha_1+\alpha_2)m_u}{m_u+m_d}$. Since the physical CP-violation angle is $\bar{\theta} = (\alpha_1 + \alpha_2)$, we finally end up with the quark mass matrix

$$M_q = \begin{pmatrix} m_u\, e^{i\bar{\theta}(\frac{m_d}{m_u+m_d})} & 0 \\ 0 & m_d\, e^{i\bar{\theta}(\frac{m_u}{m_u+m_d})} \end{pmatrix} \quad \text{and} \quad \langle\Sigma\rangle = \sigma\mathbf{1}\,. \tag{3.8}$$

Therefore, in our AdS/QCD model the strong CP-violation is easily encoded in the VEV of X as $\langle X(z)\rangle = \frac{1}{2}\left(M_q z + \langle\Sigma\rangle\, z^3\right)$ with M_q and $\langle\Sigma\rangle$ given in (3.8). This is the starting point for our holographic analysis of the strong CP-violation.

Note that we introduce strong CP-violation only through the VEV of X, while keeping the AdS/QCD theory itself, namely the various couplings and all other parameters of AdS/QCD, CP-conserving, because the QCD dynamics is CP-conserving when the angle $\bar{\theta}$ vanishes, and the angle can be absorbed into "external" quark mass parameters.

As we are interested in the effects of the strong CP violation in the first order of $\bar{\theta}$ (and $m_{u,d}$), we expand the phase exponents in M_q to get

$$M_q \simeq \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix} + i\,\bar{\theta}\,\bar{m}\,\mathbf{1} = M_q^0 + i\,\delta M_q \quad , \tag{3.9}$$

where $\bar{m}^{-1} = m_u^{-1} + m_d^{-1}$ is the reduced mass. The CP-violating axial mass $i\,\delta M_q$ is proportional to the identity matrix [9], which could also be argued from the vacuum stability [3, 4]. This in fact agrees with the previously known chiral Lagrangian with the strong CP-violation. For simplicity, we will take an isospin symmetric quark mass $m_u = m_d = m = 2\bar{m}$, then the whole $\langle X\rangle$ is proportional to the identity matrix, which we write as

$$\langle X(z)\rangle = \left[\frac{1}{2}(mz + \sigma z^3) + \frac{i}{4} m\bar{\theta}\, z\right]\mathbf{1} = v(z)\mathbf{1} = [v_0(z) + i\delta v(z)]\,\mathbf{1}\,, \tag{3.10}$$

where $v_0(z)$ is the dominant CP-conserving piece while small $i\delta v(z) = i\frac{m\bar{\theta}}{4} z$ breaks the CP symmetry.

4. The neutron electric dipole moment

The effects of the strong CP-violation in the VEV of X are generically of two kinds in the holographic models; it modifies the wave functions of 4D excitations along the fifth direction if the corresponding 5D field couples to $\langle X\rangle$, or it affects various couplings of 4D interactions after integrating over the fifth direction if the 5D coupling involves X. In consideration of the latter, we should also take into account the former as well. The CP-violating pion-nucleon coupling $\bar{g}_{\pi NN}$ that we will calculate in section 5 is such an example.

The operator (2.8) will be responsible for the electric dipole moment of the nucleons, as well as the magnetic dipole moment. This is because an axial $\mathrm{U}(1)_A$ rotation of the magnetic dipole moment is precisely the electric dipole moment, and the strong CP-violation

[9] The angle $\bar{\theta}$ should be proportional to γ_5, which is suppressed here, since we are using the chiral basis.

can be described by the non-zero $\mathrm{U}(1)_A$ phases of the quark mass matrix, so that its effect will appear as a small $\mathrm{U}(1)_A$ rotation of the magnetic dipole moment. We stress that the 5D operator (2.8) itself is CP-conserving, and we don't include explicit CP-violating 5D operator in the model. After introducing the CP-violation in $\langle X\rangle$ in the otherwise CP-conserving theory, additional CP-violating 5D operators may be generated by 5D loop effects. However, in the spirit of AdS/QCD we keep the tree-level analysis only, systematically ignoring $\frac{1}{N_c}$ corrections. Therefore, we expect our analysis captures a leading N_c contribution only except the sum rule, $d_p + d_n = 0$, whose origin goes beyond the $1/N_c$ approximation. The CP-violation introduced by $\langle X\rangle$ affects the 5D wave functions $f_{(1,2)(L,R)}$ of the (lowest) nucleon state through the mass coupling

$$\mathcal{L}_m = -g\bar{N}_1 \langle X\rangle N_2 + \text{h.c.} \quad , \tag{4.1}$$

and the resulting 4D operator of nucleons after integrating over the fifth coordinate with these wave functions will have a small CP-violating counterpart of the magnetic dipole moment, which is the electric dipole moment we are interested in.

To be explicit, the CP-violating $\langle X\rangle = v(z)\mathbf{1}$ enters the equation for the nucleon wave function $N_1 = f_{1L}(z)B_L(x) + f_{1R}(z)B_R(x)$ and $N_2 = f_{2L}(z)B_L(x) + f_{2R}(z)B_R(x)$, where $B_{L,R}(x)$ is the 4D nucleon field, as follows

$$\begin{aligned} \begin{pmatrix} \partial_z - \frac{\Delta}{z} & -g\frac{v(z)}{z} \\ -g\frac{v(z)^\dagger}{z} & \partial_z - \frac{4-\Delta}{z} \end{pmatrix} \begin{pmatrix} f_{1L} \\ f_{2L} \end{pmatrix} &= -m_N \begin{pmatrix} f_{1R} \\ f_{2R} \end{pmatrix} \\ \begin{pmatrix} \partial_z - \frac{4-\Delta}{z} & g\frac{v(z)}{z} \\ g\frac{v(z)^\dagger}{z} & \partial_z - \frac{\Delta}{z} \end{pmatrix} \begin{pmatrix} f_{1R} \\ f_{2R} \end{pmatrix} &= m_N \begin{pmatrix} f_{1L} \\ f_{2L} \end{pmatrix} \quad , \end{aligned} \tag{4.2}$$

where $\Delta = \frac{9}{2}$ is the scaling dimension of the 4D baryon operator and $m_N = 0.94\,\text{GeV}$ is the nucleon mass. To get the standard 4D kinetic term for $B(x)$, we have to normalize the eigen-functions

$$\int_0^{z_m} dz \left[\frac{1}{z^4}\left(|f_{1L}|^2 + |f_{2L}|^2\right)\right] = \int_0^{z_m} dz \left[\frac{1}{z^4}\left(|f_{1R}|^2 + |f_{2R}|^2\right)\right] = 1 \quad . \tag{4.3}$$

We are interested in the effects of small CP-violating imaginary part in $v(z) = v_0(z) + i\delta v(z)$ to linear order. The CP-conserving zeroth order eigenfunctions $f^{(0)}_{(1,2)(L,R)}$ with $v_0(z)$, obtained numerically in [14], are real functions with an important property

$$\begin{pmatrix} f^{(0)}_{1L} \\ f^{(0)}_{1R} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} f^{(0)}_{2L} \\ f^{(0)}_{2R} \end{pmatrix} \quad , \tag{4.4}$$

which can also be checked explicitly from (4.2). This is a simple consequence of 4D parity. The precise parity transformation in our model that leaves the 5D action invariant is

$$\begin{aligned} (x_0, \vec{x}, z) &\longleftrightarrow (x_0, -\vec{x}, z) \\ (A^0_L, \vec{A}_L, A^z_L) &\longleftrightarrow (A^0_R, -\vec{A}_R, A^z_R) \\ \begin{pmatrix} N_1 \\ N_2 \end{pmatrix} &\longleftrightarrow \begin{pmatrix} 0 & \gamma^0\gamma^5 \\ -\gamma^0\gamma^5 & 0 \end{pmatrix} \begin{pmatrix} N_1 \\ N_2 \end{pmatrix} . \end{aligned} \tag{4.5}$$

We see that the (lowest) nucleon state (4.4) is parity even as expected. Using this and the explicit form of $\delta v(z) = m\,\bar\theta\, z/4$, it is not difficult to find that the first order effect of $i\delta v(z)$ on the wave functions is a simple phase factor in the eigenfunctions $f_{(1,2)(L,R)}$,

$$\begin{pmatrix} f_{1L} \\ f_{2L} \end{pmatrix} = e^{i\alpha} \begin{pmatrix} f_{1L}^{(0)} \\ f_{2L}^{(0)} \end{pmatrix} \quad , \quad \begin{pmatrix} f_{1R} \\ f_{2R} \end{pmatrix} = e^{i\beta} \begin{pmatrix} f_{1R}^{(0)} \\ f_{2R}^{(0)} \end{pmatrix} \quad , \tag{4.6}$$

with

$$(\alpha - \beta) = \frac{g}{m_N} \left(\frac{\delta v(z)}{z} \right) = \frac{g}{4} \frac{m}{m_N}\, \bar\theta \quad , \tag{4.7}$$

without affecting the mass $m_N = 0.94\,\mathrm{GeV}$. Note that the common phase on f_L and f_R is not physical as it wouldn't appear in any bilinear operators $\bar N \Gamma^{MN\cdots} N$, while the above difference in phases (4.7) is a physical CP-violating effect. It is also possible to work in the frame where all wave functions are real, and the mass m_N instead gets a phase $m_N \to e^{i(\alpha-\beta)} m_N$,[10] although we will stick to the previous description for a while. We will get back to the frame of complex mass and real wave functions later, when we argue the generality of our result against other possible higher dimensional operators.

Inserting the expansion $N_1 = f_{1L}(z) B_L(x) + f_{1R}(z) B_R(x)$ and $N_2 = f_{2L}(z) B_L(x) + f_{2R}(z) B_R(x)$ into our relevant 5D operator (2.8),

$$S_{\rm dipole} = iD \int d^4x \int dz\, \sqrt{G_5}\, \left[\bar N_1 \Gamma^{MN} (F_L)_{MN} N_1 - \bar N_2 \Gamma^{MN} (F_R)_{MN} N_2 \right] , \tag{4.8}$$

it is straightforward to obtain the resulting 4D dipole moment operators. To read off the electromagnetic coupling, recall that $Q = I_3 + \cdots$ with the isospin $I_3 = (t_3^L + t_3^R)$, so that we can simply replace $A_{L,R}$ by $A_L = eA^{\rm em} t_3$ and $A_R = eA^{\rm em} t_3$ with $t_3 = \frac{1}{2}\sigma_3$ to extract couplings to the electromagnetic vector potential $A^{\rm em}$. According to the AdS/CFT correspondence, the external vector potential is simply a non-normalizable mode of the corresponding 5D gauge field, which turns out to be a simple constant mode over the fifth direction.

A quick calculation results in the following 4D dipole moment operators,[11]

$$\begin{aligned} \mathcal{L}_{\rm magnetic} &= \mu_m^{\rm ano} \left(\frac{1}{2} \bar B \sigma^{\mu\nu} \sigma_3 B \right) F_{\mu\nu}^{\rm em} \,, \\ \mathcal{L}_{\rm electric} &= d_e \left(-\frac{i}{2} \bar B \sigma^{\mu\nu} \gamma^5 \sigma_3 B \right) F_{\mu\nu}^{\rm em} \,, \end{aligned} \tag{4.9}$$

where σ_3 acts on the isospin index of the nucleon doublet $B = (p,n)^T$, and the anomalous magnetic dipole moment $\mu_m^{\rm ano}$ as well as our CP-violating electric dipole moment d_e are

[10] In the general complex mass case, the mass m_N in the first equation in (4.2) should be replaced by m_N^*.

[11] Our convention is $\sigma^{\mu\nu} = i\gamma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$ with $\gamma^\mu = \begin{pmatrix} 0 & \bar\sigma^\mu \\ \sigma^\mu & 0 \end{pmatrix}$ and $\gamma^5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. Here $\bar\sigma^\mu = (1, -\vec\sigma)$ and $\sigma^\mu = (1, \vec\sigma)$.

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given by

$$\mu_m^{\rm ano} = e\cdot(-1)\cdot(2D)\int_0^{z_m} dz\left[\frac{1}{z^3}f_{1L}^{(0)}f_{2L}^{(0)}\right],$$
$$d_e = e\cdot(\alpha-\beta)\cdot(2D)\int_0^{z_m} dz\left[\frac{1}{z^3}f_{1L}^{(0)}f_{2L}^{(0)}\right], \tag{4.10}$$

where $\frac{1}{z^3}$ factor in the integrals is traced back to $\sqrt{G_5}=\frac{1}{z^5}$ and the curved space gamma matrix carries an extra factor of z since the inverse vielbein $e_A^M = z\delta_A^M$. The coefficient D must be determined by comparing with the experiments $\mu_m^{\rm ano}=1.8\mu_N=\frac{1.8e}{2m_N}$, but for our purpose of obtaining d_e, we have a model-independent prediction for the ratio

$$\frac{d_e}{\mu_m^{\rm ano}} = -(\alpha-\beta) = -\frac{g}{4}\frac{m}{m_N}\bar\theta \quad , \tag{4.11}$$

between the electric dipole moment and the anomalous magnetic dipole moment of nucleons. This gives us

$$d_e = -1.8\mu_N\cdot\frac{g}{4}\frac{m}{m_N}\bar\theta = -\frac{1.8g}{8}\left(\frac{m}{m_N}\right)\left(\frac{e}{m_N}\right)\bar\theta. \tag{4.12}$$

Using $m_N = 0.94\,{\rm GeV}$ and ${\rm GeV}^{-1}=0.197\times10^{-13}\,{\rm cm}$, we have

$$d_e = -0.25\,g\times\left(\frac{m}{5\,{\rm MeV}}\right)\times10^{-16}\,\bar\theta \quad (e\cdot{\rm cm}), \tag{4.13}$$

in units of $(e\cdot{\rm cm})$. This is the main result of our paper.

For a model in [17] with $z_m=(330\,{\rm MeV})^{-1}$, $\sigma=(311\,{\rm MeV})^3$ and $m=2.34\,{\rm MeV}$, we have $g=9.18$ to have the correct nucleon mass eigenvalue $m_N=0.94\,{\rm GeV}$ [14], and this predicts that

$$d_e = -1.08\times10^{-16}\,\bar\theta \quad (e\cdot{\rm cm}). \tag{4.14}$$

This means that the neutron electric dipole moment is $d_n=-d_e=+1.08\times10^{-16}\,\bar\theta \quad (e\cdot{\rm cm})$.

We end this section by pointing out a new feature that our result predicts. The isospin structure in the operator (4.9) without ${\rm U}(1)_B$ component tells us that the proton electric dipole moment d_p should be equal in magnitude but minus to the neutron electric dipole moment, that is, $d_p=-d_n=d_e$. It would be very interesting to test this prediction in the future experiments, which would confirm the validity of our 5D model of nucleons.

5. CP-violating pion-nucleon coupling

We calculate the CP-violating component in the coupling of the pions to the nucleons

$$\mathcal{L}_{CPodd} = \bar g_{\pi NN}\,(\bar B\,\vec\sigma\,B)\cdot\vec\pi \quad , \tag{5.1}$$

where $B=(p,n)^T$ is the nucleon iso-doublet and $\vec\sigma$ acts on the isospin index, while the usual CP-even interaction is defined as

$$\mathcal{L}_{CPeven} = ig_{\pi NN}\,(\bar B\,\vec\sigma\,\gamma^5\,B)\cdot\vec\pi \quad . \tag{5.2}$$

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As we have the nucleon wave functions $f_{(1,2)(L,R)}$ in the previous section, what remains is to identify the pion profile along the fifth direction, so that the above 4D interactions can be read off by integrating out 5D operators over the fifth dimension. We will pay a special attention to how the strong CP-violation we introduce in $\langle X\rangle = [v_0(z) + i\delta v(z)]\mathbf{1}$ induces a mixing between the CP-even pions and the isoscalar mesons from X, since this will also contribute to (5.1) eventually.

Normally, the pions are identified as the axial $\mathrm{SU}(2)_A$ phase angles of the chiral condensate, that is, $X \sim \langle X\rangle\, e^{i\pi(x)f(z)}$ with a 5D profile $f(z)$.[12] In the AdS/QCD set-up, since we elevate the axial $\mathrm{SU}(2)_A$ to a gauge symmetry, this is no longer a gauge invariant statement, and we should consider the broken 5D $\mathrm{SU}(2)_A$ gauge field simultaneously with the phase angles of $\langle X\rangle$ which could be viewed as eaten Goldstone bosons by the (5D) Higgs mechanism. In our analysis, we choose to work in the unitary gauge, or equivalently a 5D version of the R_ξ-gauge in $\xi \to \infty$ limit, which will become clear in a moment. The advantage of this gauge-fixing is an unambiguous diagonalization of the kinetic terms of the relevant fields, which makes finding equations of motion a lot easier than other gauge choices.

A modification appearing with the strong CP-violation turns out to be that not only the phases but also the modulus of the chiral condensate takes part in the pion wave-functions; in other words, expanding $X \sim \langle X\rangle\, e^{-Q+iP}$ with P and Q being Hermitian, there is a small CP-violating mixing between the modulus Q and the $\mathrm{SU}(2)_A$ gauge field A_z in addition to the usual mixing of P and A_z.[13] (A_z is the component of the $\mathrm{SU}(2)_A$ gauge field along the fifth direction z). This implies that the resulting pion wave-function along the fifth direction involves a small Q-component as well as the dominant A_z and P part. The CP-violating Q-component in the pion wave-function will then contribute to the CP-violating coupling to the nucleons in (5.1). Of course, the operator (5.1) also receives contributions from the CP-violating phase $(\alpha-\beta)$ in the nucleon wave-functions $f_{(1,2)(L,R)}$ that we identify in the previous section.

It is rather straight-forward but tedious to perform the above mentioned procedure to obtain the 5D profile of the pions. Defining the axial vector field by $A \equiv \frac{1}{2}(A_L - A_R)$ and expanding $X = \langle X\rangle\, e^{-Q+iP} = [v_0 + i\delta v]e^{-Q+iP}$ up to the quadratic order, we have

$$\begin{aligned} S^{(2)}_{5D} = \int d^4x dz \sqrt{G_5}\,\mathrm{Tr}\bigg\{ &-\frac{1}{2g_5^2}(F_A)_{MN}(F_A)^{MN} + (v_0)^2\Big(2A_M - \partial_M P\Big)\Big(2A^M - \partial^M P\Big) \\ &+(v_0)^2(\partial_M Q)(\partial^M Q) + 2\Big((\partial_z v_0)(\partial^z v_0) + 3(v_0)^2\Big)Q^2 + 4v_0(\partial_z v_0)Q(\partial^z Q) \\ &-4\Big((\partial_z v_0)\delta v - v_0(\partial_z \delta v)\Big)Q\Big(2A^z - \partial^z P\Big)\bigg\} \quad , \end{aligned} \tag{5.3}$$

[12] This corresponds only to the phase of the chiral condensate, without affecting the current quark mass. The reason is that $f(z)$ is normalizable and decays to zero in UV region, and the IR profile of X encodes the chiral condensate, while its non-normalizable UV mode contains the current quark mass. Therefore, the normalizable pions with $f(z)$-profile do not contain the phase of the current quark mass.

[13] There does not appear any CP-violating mixing between P and the vector $\mathrm{SU}(2)_V$ since, in our set-up, $\langle X\rangle$ is proportional to identity matrix $\langle X\rangle = v(z)\mathbf{1} = [v_0(z) + i\delta v(z)]\mathbf{1}$, and the $\mathrm{SU}(2)_V$ remains unbroken. This may not hold in general for an isospin asymmetric quark mass $m_u \neq m_d$.

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where F_A is the field strength of A and the last line is the CP-violating mixing in the linear order of δv. In the above, we should remember that $\sqrt{G_5} = \frac{1}{z^5}$ and $\partial^z = g^{zz}\partial_z = -z^2\partial_z$, $A^z = -z^2 A_z$. As we are interested in the effects from the small CP-violating δv in the last line, we first solve the top line for A_z and P to find the zeroth order profile of the pion, and then treat the last line as a perturbation to calculate a small induced Q-component in the pion wave-function.

The kinetic terms are mixing among A_μ, A_z and P in the first line of (5.3), and to remove those mixing terms, it is convenient to introduce a 5D version of R_ξ gauge-fixing term [14],

$$S_{g.f.} = -\frac{1}{2\xi}\int d^4x dz\, \frac{1}{z}\left[\partial_\mu A_\mu - 2\xi\left(\frac{1}{g_5^2} z\partial_z\left(\frac{A_z}{z}\right) - \frac{2(v_0)^2}{z^2}P\right)\right]^2 \quad , \tag{5.4}$$

where everything is written in the flat metric basis, $ds^2 = \eta_{\mu\nu}dx^\mu dx^\nu - dz^2$. We then have separation between A_μ and (A_z, P), and, since the pions arise from the latter, we focus on the (A_z, P) sector only from now on. We further take $\xi \to \infty$ limit to work in the unitary gauge for simplicity. This means that we need to impose a constraint for the propagating fields in such a limit,

$$\frac{1}{g_5^2} z\partial_z\left(\frac{A_z}{z}\right) = \frac{2(v_0)^2}{z^2}P\,, \tag{5.5}$$

so that P can be replaced by A_z in the action. We then solve the resulting action for A_z to find the pion profile. The resulting relevant action is

$$\begin{aligned}\int d^4x dz \mathrm{Tr}\Bigg\{ &\frac{1}{g_5^2 z}(\partial_\mu A_z)^2 + \frac{z^3}{4g_5^4(v_0)^2}\left(\partial_\mu\partial_z\Big(\frac{A_z}{z}\Big)\right)^2 \\ &-\frac{(v_0)^2}{z^3}\left(2A_z - \partial_z\Big(\frac{z^3}{2g_5^2(v_0)^2}\partial_z\Big(\frac{A_z}{z}\Big)\Big)\right)^2 + \frac{(v_0)^2}{z^3}\Big((\partial_\mu Q)^2 - (\partial_z Q)^2\Big) \\ &-\frac{2}{z^3}\Big((\partial_z v_0)^2 - \frac{3(v_0)^2}{z^2}\Big)Q^2 - \frac{4}{z^3}v_0(\partial_z v_0)Q\partial_z Q \\ &+\frac{4}{z^3}\Big((\partial_z v_0)\delta v - v_0(\partial_z\delta v)\Big)Q\left(2A_z - \partial_z\Big(\frac{z^3}{2g_5^2(v_0)^2}\partial_z\Big(\frac{A_z}{z}\Big)\Big)\right)\Bigg\}, \end{aligned} \tag{5.6}$$

where we first solve the top line and then solve for Q from the last two lines.

The equation of motion can be easily derived from the above. As we are looking for the pion profile, we put $A_z = \pi(x)A(z)$ and $Q = \pi(x)\,q(z)$ with $q(z)$ being expected to be small and linear in δv or $\bar\theta$. The 4D pion field $\pi(x)$ satisfies $\partial_\mu\partial_\mu\pi(x) = -m_\pi^2\pi(x)$ where m_π is the pion mass that must be determined from the eigenvalue equation we will specify in a moment. The equation for $A(z)$ turns out to have a nice factorization structure, due to the underlying gauge invariance. To be explicit, defining

$$B \equiv 2A - \partial_z\Big(\frac{z^3}{2g_5^2(v_0)^2}\partial_z\Big(\frac{A}{z}\Big)\Big), \tag{5.7}$$

we find the equation for $A(z)$ becomes a equation for $B(z)$ only,

$$\partial_z\left(\frac{z^3}{(v_0)^2}\partial_z\Big(\frac{(v_0)^2}{z^3}B\Big)\right)-\frac{4g_5^2(v_0)^2}{z^2}B=-m_\pi^2 B \quad , \tag{5.8}$$

with the IR boundary condition $B(z_m)=0$. The normalizability at UV, $z\to 0$, determines the pion mass m_π as the eigenvalue of the above equation. For example, the parameters in [17], $z_m=(330\,\mathrm{MeV})^{-1}$, $g_5^2=4\pi^2$, and $v_0(z)=\frac{1}{2}(mz+\sigma z^3)$ with $m=2.34\,\mathrm{MeV}$, $\sigma=(311\,\mathrm{MeV})^3$, indeed give us $m_\pi=140\,\mathrm{MeV}$ as the lowest eigenvalue [14]. The normalization of $B(z)$ should be determined such that the 4D action of $\pi(x)$ after integrating over the z-direction must have the form

$$\int d^4x\,\mathrm{Tr}\Big[(\partial_\mu\pi)(\partial^\mu\pi)-m_\pi^2\pi^2\Big] \quad . \tag{5.9}$$

Looking at the last term in the first line of (5.6), which produces the mass term

$$-\int_0^{z_m}dz\,\frac{(v_0)^2}{z^3}B^2\cdot\int d^4x\,\mathrm{Tr}[\pi^2] \quad , \tag{5.10}$$

we see that the normalization of $B(z)$ must be fixed by

$$\int_0^{z_m}dz\,\frac{(v_0)^2}{z^3}B^2 \;=\; m_\pi^2 \quad . \tag{5.11}$$

Once we solve $B(z)$, we then find $A(z)$ via solving (5.7) with the IR boundary condition $A(z_m)=0$ and the normalizability in the UV region, $z\to 0$. Finally, $p(z)$ in $P=p(z)\pi(x)$ is determined through the constraint (5.5),

$$\frac{1}{g_5^2}z\partial_z\left(\frac{A}{z}\right)=\frac{2(v_0)^2}{z^2}p\,. \tag{5.12}$$

Having obtained the zeroth order profile $A(z)$ (and $p(z)$) for the pion, we next solve $q(z)$ in $Q=q(z)\pi(x)$ that is induced by the CP-violating mixing in the last line of (5.6). Note that the effect of this mixing to $A(z)$ and $p(z)$ is of second order in δv, and can be neglected. The equation for $q(z)$ is written as

$$\begin{aligned}\partial_z\Big(\frac{(v_0)^2}{z^3}\partial_z q\Big)+\left(\frac{m_\pi^2(v_0)^2}{z^3}-\frac{2}{z^3}\Big((\partial_z v_0)^2-\frac{3(v_0)^2}{z^2}\Big)+\partial_z\Big(\frac{2}{z^3}v_0\,\partial_z v_0\Big)\right)q\\ =-\frac{2}{z^3}\Big(\delta v\,\partial_z v_0-v_0\,\partial_z\delta v\Big)B \quad ,\end{aligned} \tag{5.13}$$

where the second line is the source for $q(z)$ induced by $\delta v(z)=\frac{m\bar\theta}{4}z$. We should impose the IR boundary condition $q(z_m)=0$ and the UV normalizability as well.

After getting the pion profile $A_z=A(z)\pi(x)$, $P=p(z)\pi(x)$ and $Q=q(z)\pi(x)$, we see that it is straightforward to insert them together with the nucleon wave-functions $f_{(1,2)(L,R)}$

[14] One can also easily check that the Gell-Mann-Oakes-Renner relation holds to a good degree in our model by calculating the pion mass for different values of the quark mass.

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into our 5D action of holographic baryons, and to read off the couplings between pions and the nucleons. Writing the resulting 4D interaction terms as

$$\mathcal{L}_{\pi NN} = i\, g^{(1)}_{\pi NN}\, (\,\bar{B}\,\vec{\sigma}\,\gamma^5\, B\,)\cdot\vec{\pi} - \frac{g^{(2)}_{\pi NN}}{2m_N}\,(\,\bar{B}\,\vec{\sigma}\,\gamma^\mu\gamma^5\, B\,)\cdot\partial_\mu\vec{\pi} + \bar{g}_{\pi NN}\,(\,\bar{B}\,\vec{\sigma}\, B\,)\cdot\vec{\pi} \quad , \tag{5.14}$$

where we expand $\pi = \vec{\pi}\cdot\frac{\vec{\sigma}}{2}$, the last term is the CP-violating pion-nucleon coupling we are interested in, with the coupling strength $\bar{g}_{\pi NN}$ given by

$$\begin{aligned}\bar{g}_{\pi NN} \;=&\; -(\alpha-\beta)\int_0^{z_m} dz\left[\frac{1}{z^4}A(z)f^{(0)}_{1L}(z)f^{(0)}_{2L}(z) + \frac{g}{2z^5}v_0(z)p(z)\Big[(f^{(0)}_{1L}(z))^2 + (f^{(0)}_{2L}(z))^2\Big]\right]\\ &+\frac{g}{2}\int_0^{z_m} dz\left[\frac{1}{z^5}\Big(v_0(z)q(z)+\delta v(z)p(z)\Big)\Big[(f^{(0)}_{1L}(z))^2 - (f^{(0)}_{2L}(z))^2\Big]\right] \quad ,\end{aligned} \tag{5.15}$$

where $(\alpha-\beta) = \frac{g}{4}\frac{m}{m_N}\bar{\theta}$ and $\delta v(z) = \frac{m\bar{\theta}}{4}z$ as given before. This is the primary result of this section. Since $q(z)$ is proportional to $\bar{\theta}$ via δv, our $\bar{g}_{\pi NN}$ is proportional to the strong CP angle $\bar{\theta}$ as expected.

For the model of $z_m = (330\,\mathrm{MeV})^{-1}$ and $v_0(z) = \frac{1}{2}(mz+\sigma z^3)$ with $m = 2.34\,\mathrm{MeV}$, $\sigma = (311\,\mathrm{MeV})^3$, we have $g = 9.18$ and our numerical result for $\bar{g}_{\pi NN}$ is

$$\bar{g}_{\pi NN} \;=\; +0.017\,\bar{\theta} \quad , \tag{5.16}$$

which is about half from the previous estimate in [4].

For completeness, we also give the expressions for the CP-conserving coupling constants $g^{(1)}_{\pi NN}$ and $g^{(2)}_{\pi NN}$,

$$\begin{aligned} g^{(1)}_{\pi NN} \;=&\; \int_0^{z_m} dz\left[\frac{1}{z^4}A(z)f^{(0)}_{1L}(z)f^{(0)}_{2L}(z) + \frac{g}{2z^5}v_0(z)p(z)\Big[(f^{(0)}_{1L}(z))^2 + (f^{(0)}_{2L}(z))^2\Big]\right] \quad ,\\ g^{(2)}_{\pi NN} \;=&\; -(2m_N)\cdot D\int_0^{z_m} dz\left[\frac{1}{z^3}A(z)\Big[(f^{(0)}_{1L}(z))^2 + (f^{(0)}_{2L}(z))^2\Big]\right] \quad .\end{aligned} \tag{5.17}$$

Note that the coupling term with $g^{(2)}_{\pi NN}$ may turn into the $g^{(1)}_{\pi NN}$-coupling by using the equation of motion for B, and the total pion-nucleon coupling is $g_{\pi NN} = g^{(1)}_{\pi NN} + g^{(2)}_{\pi NN}$. Recall also that D should be fitted against the anomalous magnetic dipole moments of the proton and neutron by (4.10),

$$(2D)\cdot\int_0^{z_m} dz\left[\frac{1}{z^3}f^{(0)}_{1L}f^{(0)}_{2L}\right] \;=\; -\frac{\mu^{\mathrm{ano}}_m}{e} \;\simeq\; -\frac{1.8}{2m_N} \quad . \tag{5.18}$$

Numerical values for the above model are $g^{(1)}_{\pi NN} = -3.64$, $g^{(2)}_{\pi NN} = -22.0$ with $g_{\pi NN} = -25.6$, which is about twice of the experiment $g^{\mathrm{exp}}_{\pi NN} = 13.6$. There is however much chance to improve several crude simplifications in constructing our model, such as the simple AdS-slicing, neglecting anomalous dimension of the three-quark baryon operators, the possibility of a different IR-cutoff, *etc.* [24].

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6. Discussion and conclusion

We have studied the CP-violating effects in strong interactions, induced by the QCD θ-term, in a bottom-up model of holographic baryons [14]. We find that the model of holographic baryons is quite useful in studying the CP-violating effects in the baryon sector such as the electric dipole moments of nucleon or the CP-violating couplings of pions to nucleons, since the CP-violating effects are easily incorporated in the bulk scalar field X, whose coupling to bulk spinors naturally leads to the calculable CP-violating effects in the baryon sector.

A Pauli term is added to the holographic model of baryons [14] to estimate the electric dipole moments of nucleons. We have fixed the coefficient of the Pauli term by fitting the anomalous magnetic moments of nucleons, though in top-down holographic models such as Sakai-Sugimoto model [25] the coefficient can be calculated reliably due to the fact that the baryons are realized as instanton solitons in holographic QCD [13, 26]. However, we stress that our result for the electric dipole moment of nucleons is model-independent, since it relies only on the ratio between the electric dipole moment and the anomalous magnetic moment of nucleons, $d_e/\mu_m^{\rm ano}$, which is independent of the coefficient of the 5D Pauli term, as shown in (4.11).

In fact, we can argue further that our result is universal against any other possible higher dimensional operators in 5D. Note that in solving (4.2), we can alternatively go to the frame where the nucleon wave-functions $f_{(1,2)(L,R)}$ are real, whereas the strong CP-violation is encoded instead in the phase of the nucleon mass $m_N \to e^{i(\alpha-\beta)}m_N$. Upon reduction to 4D, all the resulting 4D operators obtained from real wave functions $f_{(1,2)(L,R)}$, including anomalous magnetic dipole moment operator, will be CP-conserving except the nucleon mass term with the complex axial mass. Performing an axial $\mathrm{U}(1)_A$ rotation back to make the nucleon mass real, this will induce the electric dipole moment operator from the anomalous magnetic moment operator as in (4.9) with the universal prediction for the ratio

$$\frac{d_e}{\mu_m^{\rm ano}} = -(\alpha-\beta) = -\frac{g}{4}\frac{m}{m_N}\bar{\theta}\,, \tag{6.1}$$

without any regard to the details of the 5D model.

Though our model is bottom-up, the bulk spinors in our model should be interpreted as effective fields of the instanton solitons as in the top-down model, because the mesonic sector of our model, being a 5D gauge theory with a Chern-Simons term, naturally supports instanton solitons, whose Wilson line can be identified as skyrmions [27 − 29]. As shown in [13], the Pauli term therefore should not contain the $\mathrm{U}(1)$ gauge fields to reproduce the correct behavior of non-abelian instanton solitons at long distances, which leads to important sum rules for both the electric dipole moments and the anomalous magnetic moments of nucleons, which are model-independent and not subject to any $1/N_c$ corrections,

$$d_p + d_n = 0, \quad \mu_p^{\rm ano} + \mu_n^{\rm ano} = 0\,. \tag{6.2}$$

Combining our holographic estimate of the neutron electric dipole moment, given as $d_n = 1.08\times 10^{-16}\ \bar{\theta}\, e\cdot{\rm cm}$, with the current experimental bound on the neutron EDM [1],

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we obtain a limit on the θ angle reliable up to 30% coming from $1/N_c$ corrections,

$$\left|\bar{\theta}\right| < 3 \times 10^{-10}, \tag{6.3}$$

which is somewhat stronger than the previous bounds.

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